PRICING MODELS FOR OPTIONS VALUATION

A FINANCIAL MANAGEMENT PROJECT REPORT

SUBMITTED BY

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Abstract

Options are contracts that give the bearer the right, but not the obligation, to either buy or sell an amount of some underlying asset at a pre-determined price at or before the contract expires. Options are powerful because they can enhance an individual's portfolio. They do this through added income, protection, and even leverage. Options can also be used to generate recurring income. Additionally, they are often used for speculative purposes such as wagering on the direction of a stock. However, Options trading involves certain risks that the investor must be aware of before making a trade. Hence it becomes critical to valuate options and compute the associated premium for a variety of risk scenarios. This report gives a detailed analysis of three kinds of pricing models used for option valuation. Firstly, we examine an analytical model called the Black-Scholes model which garnered the 1997 Nobel Prize in Economics. Then we study the Lattice-based Binomial model and finally the simulational Monte-Carlo Model for Option Pricing. Predicting the volatility of Options trading is a daunting task and increasing complex models and techniques are required by investors while trading.

Contents

Chapter 1

Introduction

An **option** is a contract which gives the buyer (the owner or holder of the option) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price prior to or on a specified date, depending on the form of the option. Here, the **strike price** is set by reference to the **spot price** (market price) of the underlying security or commodity on the day an option is taken out, or it may be fixed at a discount or at a premium. The seller has the corresponding obligation to fulfill the transaction (to sell or buy) if the buyer "exercises" the option. There are two types of options:

- **Call Option**: An option that conveys to the owner the right to buy at a specific price
- **Put Option**: An option that conveys the right of the owner to sell at a specific price

Both types are commonly traded, but the call option is more frequently discussed [18].

The seller may grant an option to a buyer as part of another transaction, such as a share issue or as part of an employee incentive scheme, otherwise a buyer would pay a premium to the seller for the option. A call option would normally be exercised only when the strike price is below the market value of the underlying asset, while a put option would normally be exercised only when the strike price is above the market value. When an option is exercised, the cost to the buyer of the asset acquired is the strike price plus the premium, if any. When the option expiration date passes without the option being exercised, the option expires and the buyer would forfeit the premium to the seller. In any case, the premium is income to the seller, and normally a capital loss to the buyer.

Options are classified into a number of styles, the most common of which are:

- **American option**: an option that may be exercised on any trading day on or before expiration.
- **European option**: an option that may only be exercised on expiry.

Valuation of options combines a model of the behavior ("process") of the underlying price with a mathematical method which returns the premium as a function of the assumed behavior. Since it depends on a number of different variables in addition to the value of the underlying asset, options are generally complex to value. This report discusses three popular models of valuation of options, namely, the Black-Scholes Model, the Binomial Options Pricing Model, and the Monte Carlo Option Model [19].

Chapter 2

Black-Scholes Model

2.1 Introduction

The Black-Scholes model, also called the Black-Scholes-Merton model, is a mathematical model of financial derivative markets from which the Black-Scholes formula can be derived. This formula estimates the prices of call and put options. Originally, it priced European options and was the first widely adopted mathematical formula for pricing options. It has also been credited for significant increase in options trading and has great influence over modern financial pricing [20]. Prior to the invention of this formula and model, options traders didn't all use a consistent mathematical way to value options, and empirical analysis has shown that price estimates produced by this formula are close to observed prices. Today, the Black-Scholes model is widely used, though in individually modified ways, by traders and investors, as it is the fundamental strategy of hedging to best control, or "eliminate", risks associated with volatility in the assets that underlie the option [9].

In their initial formulation of the model, Fischer Black and Myron Scholes (the economists who originally formulated the model) came up with a partial differential equation known as the Black-Scholes equation [1], and later Robert Merton published a mathematical understanding of their model, using stochastic calculus that helped to formulate what became known as the Black-Scholes-Merton formula. Both Myron Scholes and Robert Merton split the 1997 Nobel Prize in Economics. Although ineligible for the prize because of his death in 1995, Fischer Black was mentioned as a contributor by the Swedish Academy [12].

In simple terms, their model determines the price of an option by calculating the return an investor gets less the amount that investor has to pay, using log-normal distribution probabilities to account for volatility in the underlying asset. The log-normal distribution of returns used in the model is based on theories of Brownian motion, with asset prices exhibiting similar behavior to the organic movement in Brownian motion [2].

2.2 Fundamental Assumptions

The Black–Scholes model assumes that the market consists of at least one risky asset, usually called the stock, and one risk-less asset, usually called the money market, cash, or bond [11].

The following assumptions holds on the assets:

- **Risk-less Rate**: The rate of return on the risk-less asset is constant and thus called the risk-free interest rate.
- **Random Walk**: The instantaneous log return of stock price is an infinitesimal random walk with drift; more precisely, the stock price follows a geometric Brownian motion, and we will assume its drift and volatility are constant (if they are time-varying, a suitably modified Black–Scholes formula can be deduced, as long as the volatility is not random).
- The stock does not pay a dividend.

The assumptions on the market are:

- No arbitrage opportunity (i.e., there is no way to make a risk-less profit).
- Ability to borrow and lend any amount, even fractional, of cash at the risk-less rate.
- Ability to buy and sell any amount, even fractional, of the stock (this includes short selling).
- The above transactions do not incur any fees or costs (i.e., friction-less market).

2.3 Notation

We use the following notation to describe the model:

- $N(\cdot)$ is the cumulative distribution function of the standard normal distribution
- $T - t$ is the time to maturity (expressed in years)
- S_t is the spot price of the underlying asset
- K is the strike price
- r is the risk-free rate (annual rate, expressed in terms of continuous compounding)
- σ is the volatility of returns of the underlying asset

2.4 Black-Scholes Formula

The Black–Scholes formula calculates the price of European put and call options. This formula is the solution of a partial differential equation representing the Black-Scholes model with the corresponding terminal and boundary conditions [8, 16]. Figure 2.1 demonstrates a European call valued using the Black-Scholes equation.

The value of a call option for a non-dividend-paying underlying stock in terms of the Black-Scholes parameters is:

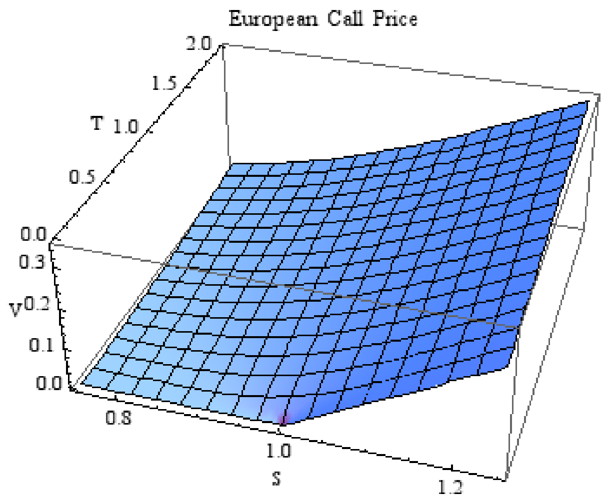

Figure 2.1: A European call valued using the Black–Scholes pricing equation for varying asset price S and time-to-expiry T. In this particular example, the strike price is set to 1.

$$
\begin{aligned}
C\left(S_t, t\right) &= N\left(d_1\right) S_t - N\left(d_2\right) K e^{-r(T-t)} \\
d_1 &= \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] \\
d_2 &= d_1 - \sigma\sqrt{T-t}
\end{aligned}
$$

The price of a corresponding put option based on put–call parity is:

$$
\begin{aligned}
P\left(S_t, t\right) &= K e^{-r(T-t)} - S_t + C\left(S_t, t\right) \\
&= N\left(-d_2\right) K e^{-r(T-t)} - N\left(-d_1\right) S_t
\end{aligned}
$$

2.5 Exemplary Problem

Consider a sample problem involving the Black-Scholes model. Let a European call option have the following parameters:

- Stock price: \$50
- Strike price: \$45
- Time to expiration: 80 days
- Risk-free interest rate: 2%
- Implied volatility: 30%

If an investor is planning to buy this call option, using the Black-Scholes model, what is the cost of this call option?

Solution. Computing,

$$d_1 = \frac{\ln \frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln \frac{50}{45} + \left(0.02 + \frac{3^2}{2}\right)\left(\frac{80}{365}\right)}{0.3\sqrt{\frac{80}{365}}} = \frac{0.105 + 0.014}{0.140} \approx 0.851$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = 0.851 - 0.3\sqrt{\frac{80}{365}} \approx 0.711$$

$N(d_1)$ and $N(d_2)$ can be found by looking at a $z-$score table:

$$N(d_1) = 0.8023$$
$$N(d_2) = 0.7611$$

Thus,

$$\begin{aligned} C(S_0, t) &= S_0 N(d_1) - Ke^{-r(T-t)} N(d_2) \\ &= (\$50 \times 0.8023) - \left(\$45 \times e^{\left(-0.02 \times \frac{50}{365}\right)} \times 0.7611\right) \\ &= \$40.12 - \$34.10 \\ &= \$6.02 \end{aligned}$$

□

In the case of this stock, there is a probability of ≈ 80% that represents the expected value at expiration, which is multiplied by \$50 to yield a \$40.12 return. The strike price is \$45 and its present value is \$44.80, which is multiplied by ≈ 76%, probability of the call option expiring in the money, to yield \$34.10.

2.6 Black Scholes Model in Practice

Nassim Nicholas Taleb, famous for his 2007 bestselling book "Black Swan" [14] which discussed unpredictable events in financial markets, along with Espen Gaarder Haug has criticized the Black-Scholes-Merton model, saying that it is "fragile to jumps and tail events" and can only handle "mild randomness." This is one of a few known challenges to the model [7]:

- **Fragility to "tail-risk" or other extreme randomness**: In general, returns do not absolutely follow a normal distribution. Market returns can be leptokurtic (or more concentrated about the mean with fat tails) which do not follow the standard model distribution.

- **The structure of Black-Scholes model doesn't reflect present realities**: The Black-Scholes model assumes a market using European call options when most options traded today are American call options that can be sold at any point. It also does not allow for dividends, something that is commonly found in options.

- **Assumption of a risk-free interest rate**: A theoretical calculation of risk-free rates is hard to come up with and, in practice, investors use proxies like the long-term yield on the US Treasury coupon bonds (generally 10-year bonds). However, this assumes that US Treasury bonds are "risk-free" when a more accurate statement would be that they're what the market assumes the least risky investment vehicles.

- **Assumption of cost-less trading**: Trading generally comes with exchange fees, the costs to buy or sell stocks and options, and the cost of time; the time it takes for the order to go through may result in changes to the price on the market. These costs can be managed, but are not included in the model.

- **Gap risk**: Also the model assumes that trading occurs continuously, unlike reality, where markets shut down for the night and then can reopen at significantly different prices to reflect new information.

One of the more common criticisms of the Black-Scholes model is the existence of a volatility smile as depicted in Figure 2.2. The Black-Scholes-Merton pricing model suggests a constant volatility and log-normal distributions of returns, where, in reality, implied volatility varies widely. Options whose strike price are said to be "deep-in-the-money" or "out-of-the-money," i.e. whose strike price is further away from the assumed underlying asset price, command higher prices than a flat volatility would suggest — their implied volatility is higher [5].

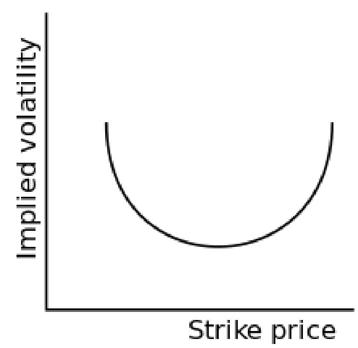

Figure 2.2: Volatility Smile

Empirically, significant pricing discrepancies between Black-Scholes model and reality have occurred more often than if returns were log-normal. Nevertheless, it is widely used in practice due to the following reasons [20]:

- Easy to calculate

- A useful approximation, particularly when analyzing the direction in which prices move when crossing critical points

- A robust basis for more refined models

- Reversible, as the model's original output, price, can be used as an input and one of the other variables solved for; the implied volatility calculated in this way is often used to quote option prices (that is, as a quoting convention).

Chapter 3

Binomial Options Pricing Model

3.1 Introduction

The Binomial Options Pricing Model (BOPM) provides a generalizable numerical method for the valuation of options. Essentially, the model uses a “discrete-time” (lattice-based) model of the varying price over time of the underlying financial instrument, addressing cases where the closed-form Black–Scholes formula is wanting. It first proposed by William Sharpe in 1978 and formalized by Cox, Ross and Rubinstein in 1979 [3] and by Rendleman and Bartter in that same year [13].

The Binomial options pricing model approach has been widely used since it is able to handle a variety of conditions for which other models cannot easily be applied. This is largely because the BOPM is based on the description of an underlying instrument over a period of time rather than a single point. As a consequence, it is used to value American options that are exercisable at any time in a given interval as well as Bermudan options that are exercisable at specific instances of time. Being relatively simple, the model is readily implementable in computer software (even in a spreadsheet like Microsoft Excel) [15].

Although computationally slower than the Black–Scholes formula, it is more accurate, particularly for longer-dated options on securities with dividend payments. For these reasons, various versions of the binomial model are widely used by practitioners in the options markets.

For options with several sources of uncertainty (like real options) and for options with complicated features (e.g., Asian options), binomial methods are less practical due to several difficulties, and Monte Carlo option models are commonly used instead. When simulating a small number of time steps Monte Carlo simulation will be more computationally time-consuming than BOPM. However, the worst-case runtime of BOPM will be $O(2^n)$, where n is the number of time steps in the simulation. Monte Carlo simulations will generally have a polynomial time complexity, and will be faster for large numbers of simulation steps. Monte Carlo simulations are also less susceptible to sampling errors, since binomial techniques use discrete time units. This becomes more true the smaller the discrete units become [15].

3.2 Binomial Option Pricing Method

The binomial pricing model traces the evolution of the option's key underlying variables in discrete-time. This is done by means of a binomial lattice (tree), for a number of time steps between the valuation and expiration dates. Each node in the lattice represents a possible price of the underlying at a given point in time [15].

Valuation is performed iteratively, starting at each of the final nodes (those that may be reached at the time of expiration), and then working backwards through the tree towards the first node (valuation date). The value computed at each stage is the value of the option at that point in time.

Option valuation using this method is, as described, a three-step process:

- Price tree generation
- Calculation of option value at each final node
- Sequential calculation of the option value at each preceding node

3.2.1 Creating binomial price tree

The tree of prices is produced by working forward from valuation date to expiration. At each step, it is assumed that the underlying instrument will move up or down by a specific factor (u or d) per step of the tree (where, by definition, $u \geq 1$ and $0 < d \leq 1$). So, if S is the current price, then in the next period the price will either be $S_{up} = S \cdot u$ or $S_{down} = S \cdot d$.

The up and down factors are calculated using the underlying volatility, σ, and the time duration of a step, t, measured in years (using the day count convention of the underlying instrument). From the condition that the variance of the log of the price is $\sigma^2 t$, we have:

$$u = e^{\sigma\sqrt{t}}$$

$$d = e^{-\sigma\sqrt{t}} = \frac{1}{u}$$

Above is the original Cox, Ross, & Rubinstein (CRR) method [3].

The CRR method ensures that the tree is recombinant, i.e. if the underlying asset moves up and then down, the price will be the same as if it had moved down and then up. Here the two paths merge or recombine. This property reduces the number of tree nodes, and thus accelerates the computation of the option price.

This property also allows that the value of the underlying asset at each node can be calculated directly via formula, and does not require that the tree be built first. The node-value will be:

$$S_n = S_0 \times u^{N_u - N_d}$$

where N_u is the number of up ticks and N_d is the number of down ticks.

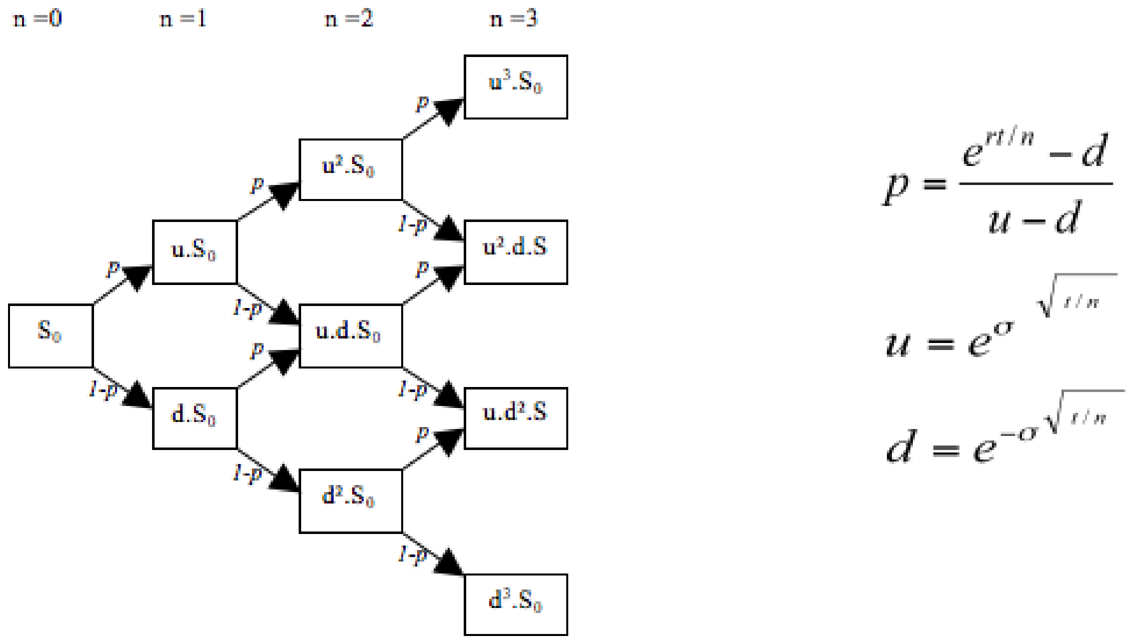

Figure 3.1: Binomial lattice for time steps between the valuation and expiration periods

3.2.2 Finding option value at each final node

At each final node of the tree (in other words, at expiration of the option) the option value is simply its intrinsic, or exercise, value:

$$\begin{aligned} &Max[(S_n - K), 0], \quad \text{for a call option} \\ &Max[(K - S_n), 0], \quad \text{for a put option} \end{aligned}$$

where K is the strike price and S_n is the spot price of the underlying asset at the n^{th} period.

3.2.3 Finding option value at earlier nodes

Once the option value is computed for each of the final nodes, it is computed for each node, starting at the penultimate time step, and working back to the first node of the tree (the valuation date) where the calculated result is the value of the option.

In general, the "binomial value" is found at each node, using the risk neutrality assumption. If exercise is permitted at the node, then the model takes the greater of binomial and exercise value at the node.

The steps are as follows:

1. Under the risk neutrality assumption, today's fair price of a derivative is equal to the expected value of its future payoff discounted by the risk-free rate. Therefore, expected value is calculated using the option values from the later two nodes (Option up and Option down) weighted by their respective probabilities. "Probability" p is ascribed to

an up move in the underlying, and "probability" $(1-p)$ of a down move. The expected value is then discounted at r, the risk free rate corresponding to the life of the option.

The following formula to compute the expectation value is applied at each node:

$$C_{t-\Delta t,i} = e^{-r\Delta t}(pC_{t,i} + (1-p)C_{t,i+1})$$

where

- $C_{t,i}$ is the option's value for the i^{th} node at time t,
- $p = \frac{e^{(r-q)\Delta t} - d}{u-d}$ is chosen such that the related binomial distribution simulates the geometric Brownian motion of the underlying stock with parameters r and σ,
- q is the dividend yield of the underlying corresponding to the life of the option.

This is pictorically represented in Figure 3.1. It follows that in a risk-neutral world futures price should have an expected growth rate of zero and therefore we can consider $q = r$ for futures.

Note that for p to be in the interval $(0, 1)$, the following condition on Δt has to be satisfied

$$\Delta t < \frac{\sigma^2}{(r-q)^2}$$

2. This result is the "Binomial Value". It represents the fair price of the derivative at a particular point in time (i.e. at each node), given the evolution in the price of the underlying to that point. It is the value of the option if it were to be held (as opposed to exercised at that point).

3. Depending on the style of the option, evaluate the possibility of early exercise at each node: if

   - the option can be exercised, and
   - the exercise value exceeds the Binomial Value

   then the value at the node is the exercise value.

   - For a European option, there is no option of early exercise, and the binomial value applies at all nodes.
   - For an American option, since the option may either be held or exercised prior to expiry, the value at each node is: Max (Binomial Value, Exercise Value).

   In calculating the value at the next time step calculated, the model must use the value selected here, for "Option up"/"Option down" as appropriate, in the formula at the node.

3.3 Exemplary Problem

Consider a stock with a strike price of \$100 with an associated call option expiring in 1 month. Assume that in the up state, this call option is worth \$10, and in the down state, it is worth \$0. An investor purchases one-half share of stock and writes or sells one call option. Let the risk-free rate (r) be 3% per year. Calculate the price of call option today using the binomial option pricing model.

Solution. It can be noted that:

- Stock price: \$100
- Stock price in one month (upstate): \$110
- Stock price in one month (downstate): \$90

The total investment today is the price of half a share less the price of the option. The possible payoffs at the end of the month are:

- Cost today (or option price): \$50
- Portfolio value (upstate): $55 - max(110 - 100, 0) = \$45$
- Portfolio value (downstate): $45 - max(90 - 100, 0) = \$45$

The portfolio payoff is equal no matter how the stock price moves. Given this outcome, assuming no arbitrage opportunities, an investor should earn the risk-free rate over the course of the month. The cost today must be equal to the payoff discounted at the risk-free rate for one month. Thus, computing the price of the call option today yields, $50 - 45e^{-rT} = \$5.11$, where $T = \frac{1}{12}$ since the call option expires in 1 month [10]. □

3.4 Relationship with Black-Scholes Model

In contrast to the Black-Scholes model, which provides a numerical result based on inputs, the binomial model allows for the calculation of the asset and the option for multiple periods along with the range of possible results for each period.

The advantage of this multi-period view is that the user can visualize the change in asset price from period to period and evaluate the option based on decisions made at different points in time. For American options, which can be exercised at any time before the expiration date, the binomial model can provide insight as to when exercising the option may be advisable and when it should be held for longer periods. By looking at the binomial tree of values, a trader can determine in advance when a decision on an exercise may occur. If the option has a positive value, there is the possibility of exercise whereas, if the option has a value less than zero, it should be held for longer periods [15].

Chapter 4

Monte Carlo Option Model

4.1 Introduction

Monte Carlo methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. The underlying concept is to use randomness to solve problems that might be deterministic in principle. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other approaches. Monte Carlo methods are mainly used in three problem classes: optimization, numerical integration, and generating draws from a probability distribution [17].

Monte Carlo methods vary, but tend to follow a particular pattern:

- Define a domain of possible inputs
- Generate inputs randomly from a probability distribution over the domain
- Perform a deterministic computation on the inputs
- Aggregate the results

4.2 Monte Carlo methods for option pricing

A Monte Carlo option model uses Monte Carlo methods to calculate the value of an option with multiple sources of uncertainty or with complicated features. The first application to option pricing was by Phelim Boyle in 1977 (for European options). In 1996, M. Boardie and P. Glasserman showed how to price Asian options by Monte Carlo. An important development was the introduction in 1996 by Carriere of Monte Carlo methods for options early exercise features [6].

Monte Carlo valuation relies on risk neutral valuation where the price of the option is its discounted expected value. The technique applied then follows as:

1. Generate a large number of possible, but random, price paths for the underlying via simulation.
2. Calculate the associated exercise value (payoff) of the option for each path.

3. Average the payoffs.

4. Discount to today, obtaining the value of the option.

Thus, we can model an option on equity with one source of uncertainty: the price of the underlying stock S_t such that it follows a geometric Brownian motion with constant drift μ and volatility σ. Hence,

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where dW_t is found via a random sampling from a normal distribution. Since the underlying random process is the same, for enough price paths, the value of a european option here should be the same as under Black Scholes. More generally though, simulation is employed for path dependent exotic derivatives, such as Asian options where the payoff is determined by the average underlying price over some pre-set period of time, thus reducing the risk of market manipulation of the underlying instrument at maturity [17].

4.3 Least Square Monte Carlo

Least Square Monte Carlo is a technique for valuing early-exercise options (i.e. Bermudan or American options). It was first introduced by Jacques Carriere in 1996. It is based on the iteration of a two step procedure:

- First, a backward induction process is performed in which a value is recursively assigned to every state at every timestep. The value is defined as the least squares regression against market price of the option value at that state and time-step. Option value for this regression is defined as the value of exercise possibilities (dependent on market price) plus the value of the timestep value which that exercise would result in (defined in the previous step of the process).

- Secondly, when all states are valued for every timestep, the value of the option is calculated by moving through the timesteps and states by making an optimal decision on option exercise at every step on the hand of a price path and the value of the state that would result in. This second step can be done with multiple price paths to add a stochastic effect to the procedure [17].

4.4 Exemplary Simulation Problem

Consider calculating the price of a call option using Monte-Carlo Simulation in Microsoft Excel where

- Stock price (S): \$195

- Exercise price (X): \$200

- Risk-free rate (r_f): 5%

- Standard deviation of returns (σ): 0.3

- Volatility (s): 30%
- Time to expiry (Δt): 0.25

Solution. The role of Monte Carlo simulation is to generate several future value of the stock based on which we can calculate the future value of the call option.

1. The changes in the stock prices can be calculated using the formula

$$\Delta S = Sr_f \Delta t + S\sigma\varepsilon\sqrt{\Delta t}$$

   The first term is a **drift** and the second term is a **shock**. For each time period, the model assumes that the price will **drift** up by the expected return. But the drift will be shocked (added or subtracted) by a random shock. Here, ε represents the random number generated from a standard normal probability distribution. In Excel, this number can be computed using the $Rand()$ function. For the purpose of this example, we will generate 1000 random paths, however the higher the number of trials, the more accurate the result obtained. If $\varepsilon = 0.576548$, $\Delta S = 19.29978$.

2. Calculate the future value of the stock price ($S + \Delta S$) for each path. The stock value at expiry will be $195 + 19.29978 = 214.29978$

3. The option value at expiry is given by $= MAX(0, S - X) = MAX(214.29978 - 200) =$ \$14.29978. The above three steps will be repeated 1000 times to get 1000 option values.

4. Now, the average of the computed 1000 option values is taken which is $\approx$ \$9.671. It can be noted that this value will change every time the spreadsheet is recalculated.

5. The option value will be discounted to the present value by multiplying it with e^{-rt} which is approximately \$9.95.

Upon comparing the result to the results from a Black-Scholes calculator, it can be noted that the results will get closer upon increasing the number of trials [4].

□

4.5 Conclusion

Monte Carlo Methods are particularly useful in the valuation of options with multiple sources of uncertainty or with complicated features, which would make them difficult to value through Black-Scholes method or lattice-based computation. The technique is thus widely used in valuing path dependent structures like lookback and Asian options and in real options analysis. Additionally, as above, the modeller is not limited as to the probability distribution assumed. Monte Carlo methods are usually too slow to be competitive against other analytic methods and thus are a method of last resort. With faster computing capability this computational constraint is less of a concern.

Chapter 5

Conclusion

We have discussed three popular models (the Black-Scholes Model, the Binomial Options Pricing Model, and the Monte Carlo Option Model) for valuation of options with an exemplary example each. The Black-Scholes Model provides closed form analytical solutions, The Binomial Option pricing Model measures the dynamics of the option's theoretical value for discrete time intervals over the option's life, thus approximating the theoretical value produced by Black-Scholes, to arbitrary degrees of precision. Finally, the Monte Carlo Option Model provides a simulational solution useful in conditions of multiple uncertainty sources or options with complicated features.

After the financial crisis of 2007–2008, valuation of options became complicated with the entry of counterparty credit risk considerations, previously performed in an entirely "risk neutral world" [19]. There are then three major developments regarding option pricing:

- For discounting, the overnight indexed swap (OIS) curve is now typically used for the "risk free rate", as opposed to LIBOR as previously.
- Option pricing must consider the volatility surface, and the numerics will then require a zeroth calibration step, such that observed prices are returned before new prices can be calculated. To do so, banks will apply local or stochastic volatility models, such as Heston model.
- The risk neutral value, no matter how determined, is then adjusted for the impact of counterparty credit risk via a Credit Valuation Adjustment (CVA).

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